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Superselection Rules from Measurement Theory

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Abstract

In quantum theory, physically measurable quantities of a microscopic system are represented by self-adjoint operators. However, not all of the self-adjoint operators correspond to measurable quantities. The superselection rule is a criterion to distinguish measurable quantities. Any measurable quantity must obey the superselection rules. By contraposition, any quantity which does not obey the superselection rules cannot be measured. Although some of superselection rules were proved, the raizon d'être of the superselection rules has been still obscure. In this paper we deduce the superselection rules from an assumption on symmetry property of measurement process. We introduce the notion of covariant indicator, which is a macroscopic observable whose value indicates the value of a microscopic object observable. We prove that if an object system has a quantity that is conserved during the measurement process, other quantities that do not commute with the conserved quantity are non-measurable by the covariant indicator. Our derivation of superselection rules is compared with the uncertainty relation under the restriction by a conservation law. An implication of the color superselection rule for the color confinement is discussed. It is also argued that spontaneous symmetry breaking enables a measurement that the superselection rule prohibits.

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1 Introduction

In the standard formulation of quantum mechanics, a state of a physical system is represented by a unit ray in a Hilbert space and an observable quantity is represented by a self-adjoint operator acting on the Hilbert space. However, it is known that not all of self-adjoint operators in realistic models correspond to measurable physical quantities. Although the notion of measurability will be precisely defined later in this paper, here measurability means ability for making correlation between a microscopic quantity to be measured and a macroscopic quantity to be read out directly. If a variation of the value of the microscopic quantity causes a change of the value of the macroscopic quantity via interaction between the microscopic and the macroscopic systems, we say that we can measure the microscopic quantity. For example, Millikan determined electric charges of electrons by measuring velocities of charged oil droplets suspended between two metal electrodes in the gravitational field. In this case, the electron charge causes a change of motion of the oil droplet, and he inferred the electron charge from the data on the motion of the oil droplet.

In the context of quantum field theory, the electric current $J^\mu = \bar{\psi}\gamma^\mu\psi$ is defined in terms of the Dirac spinor field operator ψ for electrons. The electric current J^μ is self-adjoint and measurable. However, the operators

$$\frac{1}{2}(\psi + \psi^\dagger), \qquad \frac{1}{2i}(\psi - \psi^\dagger) \tag{1.1}$$

are self-adjoint but they are not measurable even via indirect methods.

Around 1950 it was puzzling physicists that the intrinsic parity transformation of spinor field was not uniquely defined. A phase factor can be multiplied on the parity-transformed spinor field and it is not uniquely fixed. Wick, Wightman, and Wigner [1] noticed that the ambiguity in the definition of the parity transformation is allowed since *the spinor field itself is not measurable*. Thus any choice among possible phase factors does not make changes in predictions that can be tested by experiments.

From their argument physicists have learned that not every self-adjoint operator appearing in the formulation of quantum theory corresponds to a physically measurable quantity. Hence, we would like to have a criterion with which we can select measurable operators among all the self-adjoint operators. Superselection rules work as such criteria.

A superselection rule is stated as follows. There is an operator J, which is called the superselection charge. If a self-adjoint operator A represents a measurable quantity, it must satisfy the commutativity

$$[J, A] = 0. \tag{1.2}$$

This is a superselection rule, which is a necessary condition for measurability of A.

The superselection rule can be compared with a conservation law. The conservation of J is formulated as

$$[J, H] = 0, \tag{1.3}$$

where H is the Hamiltonian H of the system. The conservation law (1.3) requires that J commutes with the Hamiltonian H while the superselection rule (1.2) requires that J commutes with all of the measurable quantities. Thus the superselection rule is a stronger requirement for J than the conservation law. It can be said that the superselection rule is an extreme form of conservation laws.

The history of studies of superselection rules has been reviewed by Wightman [2] in detail. Here we briefly review the development of studies of superselection rules not necessarily in chronological order. Wick, Wightman, and Wigner [1] noticed that the fields of half-integer spins are non-measurable and they formulated the univalence superselection rule, which forbids measurements of half-integer spin fields. In their formulation the superselection charge is $J = (-1)^{2j}$, where j is the total angular momentum of the system. First they proved the univalence superselection rule using the time reversal symmetry. Later Hegerfeldt, Kraus, and Wigner [3] proved it using the rotational symmetry and they identified the superselection charge as $J = R(2\pi)$, a rotation by a 2π angle around any axis. Wick, Wightman, and Wigner [1] suggested that the electric charge could be another superselection charge. Later Strocchi and Wightman [4] proved it in the context of quantum electrodynamics. In general, the Hilbert space of a system is decomposed into subspaces which belong to distinct eigenvalues of a superselection charge and each subspace is called a sector. Ojima [5] refined the notion of sector as a quasiequivalence class of factor states of the algebra of measurable quantities. No measurable quantity has nonvanishing matrix elements between arbitrary two state vectors that belong to different sectors. Hence the relative phase of two vectors in different sectors is un-observable and the superposition of the two vectors looks like a mixed state for any measurements. This kind of apparent loss of coherence of superposed state vectors is called decoherence by Zurek [6, 7, 8]. He noted that superselection rules can provide a mechanism which generates classical behavior from quantum physics. Machida and Namiki [9] discussed the mechanism of reduction of wave packet and Araki [10] showed that their theory can be formulated in terms of superselection charges that have continuous spectra. Doplicher, Haag, and Roberts [11] showed that an abelian group of superselection charges can be reconstructed from the algebra of observable quantities. This argument has been extended to include non-abelian groups [12, 13]. They also showed that the fermionic fields can be reconstructed from the algebra of observable bosonic quantities [14]. On the other hand, the spectra of superselection charges are not changed by operations of microscopic local observables and hence they play roles of macroscopic classical variables or order parameters, which label distinct sectors. This aspect of superselection charges has been noted by Wightman [2], Sewell [15], and Ojima [16]. Even though Hepp [17] did not use the word "superselection rule," he showed that disjoint representations of a local observable algebra are parameterized by expectation values of macroscopic observables, which are equivalent to the superselection charges. Ojima [5] proposed the Micro-Macro duality, which signifies bi-directional functions between the category of microscopic quantum systems and the category of macroscopic classical systems.

In the view of the Micro-Macro duality, a macroscopic system emerges from a microscopic system while the macroscopic system works as a describer, an interpreter and a controller acting on the microscopic system. In this picture, the superselection sector plays a role of border between microscopic physics and macroscopic physics.

The history of measurement theory is too huge for reviewing. Here we only mention Ozawa's theory [18], which axiomatically characterizes physically feasible measurement processes and proves that all feasible measurements can be described by models of the von Neumann type. Hence, we can use von Neumann models without excluding other possibilities. On the other hand, in the theory of measurement, it had been a subtle problem to define equality of two observables which belong to distinct subsystems or equality of two observables which are defined at different times. Ozawa [19] proposed saying that two observables have perfect correlation if the joint probability distribution of outcomes of simultaneous measurements of the two observables is well defined and moreover if the probability for obtaining different outcomes of the two observable is zero. In this paper we call the perfect correlation Ozawa equality.

Let us turn our attention to the subject of this paper. Superselection rules often take forms of forbidding rules. For example, a quantity that is variant under 2π rotation must not be measured, and a gauge variant quantity must not be measured. However, the reason why those measurements are impossible is still vague. The purpose of this paper is to explain a mechanism that makes those measurements impossible. We deduce a general superselection rule as a consequence of symmetries of measurement processes. More concretely, we derive the superselection rule (1.2) for arbitrary measurable quantities from the conservation law (1.3) for the superselection charge.

In this paper, we will begin our discussion by examining simple examples and show that the conservation law of the momentum of an object system prevents the position measurement. For formulating the general problem, we will introduce three notions; isolated conservation law, covariant indicator, and Ozawa equality. Using these notions, we will prove the main theorem; only a quantity that commutes with the isolated conserved quantity is measurable by a covariant indicator. This is the most general form of the superselection rules. We will compare this result with the Wigner-Araki-Yanase-Ozawa theorem. We will also discuss implications of superselection rules for both abelian and non-abelian gauge symmetries. This discussion may give an insight for understanding of color confinement.

2 Preliminary studies

2.1 Von Neumann model of position measurement

To see an example in which a measurement and a conservation law are incompatible, let us investigate the von Neumann model of position measurement [18, 20]. Assume that we have two systems; one is a microscopic object to be observed and the other is a measuring apparatus. The object system has a pair of canonical variables (q, p) and the apparatus has

another pair of canonical variables (Q, P). Suppose that we aim to measure the position q of the object by reading the position Q of the indicator. In the von Neumann model, time evolution of the composite system is described by the Hamiltonian

$$H_N := KqP \tag{2.1}$$

with a coupling constant K. The von Neumann Hamiltonian H_N does not have kinetic terms of the respective subsystems but has only their interaction term. We take the Heisenberg picture in which the operators change via time evolution while the state vectors remain unchanged. Accordingly, the indicator moves as

$$Q \mapsto \alpha(Q) := e^{iH_N t/\hbar}\, Q\, e^{-iH_N t/\hbar} = Q + q \tag{2.2}$$

when $Kt/\hbar = 1$. Here used the canonical commutation relation $[Q, P] = i\hbar$. Hence, if we know the initial position distribution of Q, we can infer the object position q by reading the indicator position $\alpha(Q)$ after the measurement interaction. It is to be noted that a mechanism correlating the indicator position to the object position is necessary for accomplishing a meaningful measurement.

On the other hand, the momentum of the object system changes to

$$p \mapsto \alpha(p) := e^{iH_N t/\hbar}\, p\, e^{-iH_N t/\hbar} = p - P \tag{2.3}$$

after the measurement interaction. Here used $[q, p] = i\hbar$. Even if we replace H_N by another Hamiltonian H to define a more general model, the Hamiltonian H must contain the operator q to make correlation between q and $\alpha(Q)$. Hence, H does not commute with p and causes a change of the momentum as $e^{iHt/\hbar}\, p\, e^{-iHt/\hbar} \neq p$. Change of the object momentum is unavoidable in any position measurement. As a contraposition, we can say that *we cannot measure the object position q with conserving the object momentum p.* The position measurement and the momentum conservation are incompatible. Here is a hint for understanding the general superselection rules.

2.2 Conservation vs. measurement

Let us investigate another model which exemplifies incompatibility between conservation and measurement of noncommutative observables. Assume that the object system consists of n massive particles in the one-dimensional space. The mass, position, and momentum of each particle are denoted as m_r, q_r, p_r $(r = 1, \cdots, n)$. The center of mass and the total momentum of the object system are defined as

$$x := \frac{\sum_{r=1}^{n} m_r q_r}{\sum_{r=1}^{n} m_r}, \qquad p_x := \sum_{r=1}^{n} p_r, \tag{2.4}$$

respectively. The measuring apparatus has an observable M, which is called a meter, a pointer, or an indicator. We would like to design a measurement process that causes a shift of the meter as

$$M \mapsto \alpha(M) := e^{iHt/\hbar}\, M\, e^{-iHt/\hbar} = M + x \tag{2.5}$$

at a specific time t. Moreover, it is required that the total momentum of the object system is conserved as

$$p_x \mapsto \alpha(p_x) := e^{iHt/\hbar}\, p_x\, e^{-iHt/\hbar} = p_x. \tag{2.6}$$

Then, it is easily proved that there is no measurement process α satisfying both the shift property (2.5) and the conservation law (2.6). Since the momentum p_x is a physical quantity attributed to the object system and the meter observable M is attributed to the apparatus, they commutes, $[p_x, M] = 0$. The mapping α describing the time evolution of any physical quantity $A \mapsto \alpha(A) = e^{iHt/\hbar}\, A\, e^{-iHt/\hbar}$ is an automorphism of the algebra of observables.[2] Hence,

$$[\alpha(p_x), \alpha(M)] = \alpha([p_x, M]) = 0. \tag{2.7}$$

On the other hand, the center of mass x and the total momentum p_x satisfy the canonical commutation relation $[x, p_x] = i\hbar$. Therefore, the two assumptions (2.5) and (2.6) imply that

$$[\alpha(p_x), \alpha(M)] = [p_x, M + x] = -i\hbar, \tag{2.8}$$

which contradicts (2.7). Hence, there is no Hamiltonian satisfying the two requirements (2.5) and (2.6).

Although the requirement (2.5) may be replaced by a more relaxed requirement, the consequence remains unchanged. We may use another Hamiltonian H to define the time evolution $\alpha(M) = e^{iHt/\hbar}\, M\, e^{-iHt/\hbar}$. Instead of (2.5), we require that the meter observable M changes to $\alpha(M) = f(M, x)$, a nontrivial function of x, via the measurement process. In this case, again we have $[\alpha(p_x), \alpha(M)] = \alpha([p_x, M]) = 0$. On the other hand, we have $[\alpha(p_x), \alpha(M)] = [p_x, f(M, x)] \neq 0$ for any nontrivial function $f(x)$. Thus it is impossible to design a measurement process α satisfying both the momentum conservation $\alpha(p_x) = p_x$ and the meter shift condition $\alpha(M) = f(M, x)$. We conclude that *any process cannot make a correlation between the center-of-mass position of the object system and the meter position of the apparatus without violating conservation of the total momentum of the object system.*

The above argument can be generalized for any quantity A of the object system. We would like to have a measurement process that causes the shift of the meter as $M \mapsto \alpha(M) = M + A$. The momentum conservation implies $[p_x, H] = 0$ and $\alpha(p_x) = e^{iHt/\hbar}\, p_x\, e^{-iHt/\hbar} = p_x$. Since observables belonging to different subsystems commute, we have $[p_x, M] = 0$. These yield the relation

$$0 = \alpha([p_x, M]) = [\alpha(p_x), \alpha(M)] = [p_x, M + A] = [p_x, A]. \tag{2.9}$$

Hence we reach the consequence that any measurable quantity A must satisfy

$$[p_x, A] = 0. \tag{2.10}$$

[2] In the algebraic formalism of quantum theory, the algebra of observables can contain both self-adjoint operators and non-self-adjoint operators. Since a product of two self-adjoint operators is usually non-self-adjoint, it is convenient to accept non-self-adjoint elements into the algebra. A genuine 'observable' quantity is demanded to be self-adjoint.

Thus we deduced a superselection rule from the momentum conservation law. Any quantity that does not commute with p_x cannot be measured via a momentum-conserving process. By generalizing this argument, we can derive the superselection rule $[J, A] = 0$ of (1.2) from the conservation law $[J, H] = 0$ of (1.3). This generalization will be established as the main theorem of this paper.

For the momentum superselection rule (2.10), the relative coordinate $q_s - q_r$ of two particles commutes with the total momentum p_x. Hence, $q_s - q_r$ is measurable. In a three-particle system,

$$A = \frac{m_1 q_1 + m_2 q_2}{m_1 + m_2} - q_3 \tag{2.11}$$

commutes with both $p_1 + p_2 + p_3$ and $m_2 p_1 - m_1 p_2$. Hence the quantity A is also measurable via a momentum-conserving process.

In the above argument, it is required that the total momentum of the object particles alone is conserved. On the other hand, in a usual argument the momentum conservation means that the sum of the momenta of the object particles and the momentum of another system interacting with the particles is constant in time duration. In this paper we propose to call the conservation of a quantity attributed to only the object system an isolated conservation law.

Conservation laws are related to symmetry properties of physical systems. The analysis presented above suggests that there is a relation between the symmetry of the measurement process and the superselection rule. We will further investigate this point in the following.

3 Formulation of the problem and the main theorem

3.1 Definitions of basic notions

Here we introduce three notions: isolated conservation law, covariant indicator, and Ozawa equality to describe the problem under consideration. We will deduce the superselection rules using these notions.

Isolated conservation law

We use the concept of group action for characterizing correlation between distinct systems. Suppose that we have two systems, an object system and a measuring apparatus. The object system has an algebra $\mathscr{A}$ of its physical quantities and the apparatus has an algebra $\mathscr{M}$ of its physical quantities. The automorphism group of the algebra $\mathscr{A}$ is denoted as $\mathrm{Aut}(\mathscr{A})$. Similarly, the automorphism group of the algebra $\mathscr{M}$ is denoted as $\mathrm{Aut}(\mathscr{M})$. Let a group G act on the two systems. For making our argument mathematically rigorous, it is safer to assume that the group G is a compact Lie group. The group action on each system is described by a group homomorphism $\sigma : G \to \mathrm{Aut}(\mathscr{A})$, $g \mapsto \sigma_g$ and a group homomorphism $\tau : G \to \mathrm{Aut}(\mathscr{M})$, $g \mapsto \tau_g$. We can construct tensor products $\sigma_g \otimes \tau_g$, $\sigma_g \otimes \mathrm{id}$, $\mathrm{id} \otimes \tau_g$; all of them are automorphisms of the tensor product algebra $\mathscr{A} \otimes \mathscr{M}$ of the

composite system. A measurement process is described by the time-evolution automorphism $\alpha : \mathscr{A} \otimes \mathscr{M} \to \mathscr{A} \otimes \mathscr{M}$. If the diagram

$$\begin{array}{ccc} \mathscr{A} \otimes \mathscr{M} & \xrightarrow{\alpha} & \mathscr{A} \otimes \mathscr{M} \\ {\scriptstyle \sigma_g \otimes \tau_g}\downarrow & & \downarrow{\scriptstyle \sigma_g \otimes \tau_g} \\ \mathscr{A} \otimes \mathscr{M} & \xrightarrow{\alpha} & \mathscr{A} \otimes \mathscr{M} \end{array} \tag{3.1}$$

is commutative for arbitrary $g \in G$, we say that the measurement process α is G-invariant or that the measurement process admits the *total conservation law* associated to the action $\sigma \otimes \tau$ of the group G. The commutativity of the diagram (3.1) means that the equation

$$\alpha((\sigma_g \otimes \tau_g)(B)) = (\sigma_g \otimes \tau_g)(\alpha(B)) \tag{3.2}$$

holds for an arbitrary physical quantity $B \in \mathscr{A} \otimes \mathscr{M}$ and for an arbitrary group element $g \in G$.

The above definition of the G-invariance of the measurement process α relies on the algebraic formalism of quantum mechanics [21]. We can reformulate it in the familiar operator formalism as shown below. Suppose that the group G has generators $J \in \mathscr{A}$ and $K \in \mathscr{M}$. Then the group actions are implemented by unitary transformations as

$$\sigma_s(A) = e^{iJs/\hbar} A \, e^{-iJs/\hbar}, \qquad \tau_s(M) = e^{iKs/\hbar} M \, e^{-iKs/\hbar} \tag{3.3}$$

for arbitrary $s \in \mathbb{R}$, $A \in \mathscr{A}$, $M \in \mathscr{M}$. On the other hand, the time evolution is described by the Heisenberg operator

$$\alpha_t(B) = e^{iHt/\hbar} \, B \, e^{-iHt/\hbar}, \tag{3.4}$$

where H is the Hamiltonian of the composite system and $B \in \mathscr{A} \otimes \mathscr{M}$ is an arbitrary physical quantity of the composite system. If the sum $J + K$ commutes with H as

$$[(J + K), H] = 0, \tag{3.5}$$

then $J + K$ satisfies the conservation law $\alpha_t(J + K) = J + K$. Therefore, the equality

$$\begin{aligned} \alpha_t((\sigma_s \otimes \tau_s)(B)) &= e^{iHt/\hbar} \, e^{i(J+K)s/\hbar} \, B \, e^{-i(J+K)s/\hbar} \, e^{-iHt/\hbar} \\ &= e^{i(J+K)s/\hbar} \, e^{iHt/\hbar} \, B \, e^{-iHt/\hbar} \, e^{-i(J+K)s/\hbar} \\ &= (\sigma_s \otimes \tau_s)(\alpha_t(B)) \end{aligned} \tag{3.6}$$

holds for an arbitrary $B \in \mathscr{A} \otimes \mathscr{M}$ and for arbitrary $s, t \in \mathbb{R}$. Thus the commutativity (3.2) is ensured.

If, instead of (3.1), the diagram

$$\begin{array}{ccc} \mathscr{A} \otimes \mathscr{M} & \xrightarrow{\alpha} & \mathscr{A} \otimes \mathscr{M} \\ {\scriptstyle \sigma_g \otimes \mathrm{id}}\downarrow & & \downarrow{\scriptstyle \sigma_g \otimes \mathrm{id}} \\ \mathscr{A} \otimes \mathscr{M} & \xrightarrow{\alpha} & \mathscr{A} \otimes \mathscr{M} \end{array} \tag{3.7}$$

is commutative for arbitrary $g \in G$, we say that the measurement process α admits the *isolated conservation law* associated to the action σ of the group G. If the conservation law of J, that is

$$[J, H] = 0, \tag{3.8}$$

holds, the commutativity $\alpha_t((\sigma_s \otimes \mathrm{id})(B)) = (\sigma_s \otimes \mathrm{id})(\alpha_t(B))$ is verified by a calculation similar to (3.6). It is to be noted that the quantity J is attributed to the object system only. In this case the conserved quantity J becomes the superselection charge as seen below.

Although we introduced the operators H, J, K for making the formulation familiar to physicists and for showing the conserved quantities explicitly, we will not use them but later we will use only the algebraic relation (3.7) for deriving the superselection rules.

Covariant indicator

We have a composite system of the object and the apparatus. Any quantity B changes as $B \mapsto \alpha(B) = e^{iHt/\hbar} B e^{-iHt/\hbar}$ via a measurement process. In general, we read out the meter observable $\alpha(M)$ of the apparatus after the measurement process and infer the value of the object observable A. To perform a meaningful measurement we need to make correlation between the initial object quantity A and the indicator $\alpha(M)$. In other words, a change of A should be followed by a change of $\alpha(M)$. Hence, it is appropriate to characterize their correlation by the covariance of A and $\alpha(M)$ under group transformations.

Let us examine how the covariance is formulated in the von Neumann model of position measurement. In that model a shift of the object position by a length $b \in \mathbb{R}$ is described as

$$\sigma_b(q) := e^{ipb/\hbar}\, q\, e^{-ipb/\hbar} = q + b. \tag{3.9}$$

A shift of the indicator position is similarly described as

$$\tau_b(Q) := e^{iPb/\hbar}\, Q\, e^{-iPb/\hbar} = Q + b. \tag{3.10}$$

In this situation, the covariance of the object and the indicator is characterized by the condition

$$\sigma_b(\alpha(Q)) = \alpha(\tau_b(Q)), \tag{3.11}$$

which is verified as

$$\begin{aligned}
\sigma_b(\alpha(Q)) &= e^{ipb/\hbar}\, e^{iH_N t/\hbar}\, Q\, e^{-iH_N t/\hbar}\, e^{-ipb/\hbar} \\
&= e^{ipb/\hbar}(Q + q)\, e^{-ipb/\hbar} \\
&= Q + (q + b) \\
&= (Q + q) + b \\
&= e^{iH_N t/\hbar}(Q + b)\, e^{-iH_N t/\hbar} \\
&= e^{iH_N t/\hbar}\, e^{iPb/\hbar}\, Q\, e^{-iPb/\hbar}\, e^{-iH_N t/\hbar} \\
&= \alpha(\tau_b(Q))
\end{aligned} \tag{3.12}$$

for the von Neumann Hamiltonian (2.1). Thus, the shift of the indicator follows the shift of the initial position of the object.

By generalizing the above consideration, we define the notion of a meter observable moving covariantly to the object. If the meter observable M of the apparatus satisfies the commutative diagram

$$\begin{tikzcd} \mathbf{1}\otimes M \arrow[r, "\alpha"] \arrow[d, "\mathrm{id}\otimes\tau_{\mathrm{g}}"'] & \alpha(\mathbf{1}\otimes M) \arrow[d, "\sigma_g\otimes\mathrm{id}"] \\ \mathbf{1}\otimes\tau_g M \arrow[r, "\alpha"] & \alpha(\mathbf{1}\otimes\tau_g M)=\sigma_g(\alpha(\mathbf{1}\otimes M)) \end{tikzcd} \tag{3.13}$$

for arbitrary $g\in G$ and for the identity $\mathbf{1}\in\mathscr{A}$, then M is called a *G-covariant indicator*. This is a generalization of the shift-covariance property (3.11) of the meter.

The notion of covariance of observables under group actions has been introduced by Holevo [22]. However, our definition of covariance is different from his. In Holevo's definition, the covariance means a group transformation property of a probability operator-valued measure (POVM) of a single object system. In our definition, the covariance means the correlation of group transformation properties of two systems.

Ozawa equality

In measurement theory, it had been a subtle issue to define equality of two observables belonging to distinct subsystems. We need to compare A and $\alpha(M)$; A is an object observable before the measurement process while $\alpha(M)$ is a meter observable after the process. Naively, it seems necessary to make the operator identity $A=\alpha(M)$ for carrying out a precise measurement. However, requiring them to be equal without depending on the state of the system is an excessive demand. Once the initial state of the composite system is prepared, a some part of the spectrum of an observable is realized as measurement outcomes, but not all of the spectral values are realized as outcomes with nonzero probability. Even if some parts of the spectra of A and $\alpha(M)$ are different, if their realization probabilities are zero, we do not see their difference. For saying that A and $\alpha(M)$ are *equal in measurements*, it is necessary and sufficient that their spectral values appearing with nonzero probability coincide.

Ozawa [19] has formulated the notion of perfect correlation that characterizes the equality of two observables in measurement. Suppose that two self-adjoint operators A and B on a Hilbert space $\mathscr{H}$ have spectral decompositions

$$A=\int\lambda\,E^A(d\lambda),\qquad B=\int\lambda\,E^B(d\lambda) \tag{3.14}$$

with their respective projection measures E^A and E^B. It is said that two observables A and B are *perfectly correlated in a state $\psi\in\mathscr{H}$* if the equation

$$E^A(\Delta)\psi=E^B(\Delta)\psi \tag{3.15}$$

holds for an arbitrary Borel subset $\Delta \subset \mathbb{R}$. Suppose that this relation (3.15) holds and that the measurements of A and B are performed on the state ψ. When the outcome of A is in the range Δ, the outcome of B is also in Δ, and vice versa. This property justifies calling the relation (3.15) the *perfect correlation of* A *and* B *in* ψ. This relation is denoted as

$$A \equiv_\psi B. \tag{3.16}$$

Ozawa proved that the perfect correlations satisfy (i) the reflexive law: $A \equiv_\psi A$, (ii) the symmetric law: $A \equiv_\psi B \Rightarrow B \equiv_\psi A$, (iii) the transitive law: $A \equiv_\psi B$, $B \equiv_\psi C \Rightarrow A \equiv_\psi C$. Hence the perfect correlation is an equivalence relation. In this paper we call it *Ozawa equality.*

This equality can be expressed in terms of the GNS construction [23] (GNS is an abbreviation for Gel'fand-Naĭmark-Segal). A state ω associated to the vector $\psi \in \mathscr{H}$ is a linear functional

$$\omega(A) := \langle\psi|A|\psi\rangle \tag{3.17}$$

for $A \in \boldsymbol{B}(\mathscr{H})$, that is the set of all bounded operators on $\mathscr{H}$. Restricting the state ω on the algebra $\mathscr{G}(A, B)$ generated by A and B, and using the GNS procedure, we can construct a representation π_ω of the algebra $\mathscr{G}(A, B)$. Ozawa himself proved [19] that the perfect correlation (3.15) is equivalent to the equality of the GNS-representing operators

$$\pi_\omega(A) = \pi_\omega(B). \tag{3.18}$$

For becoming familiar with the idea of Ozawa equality, let us examine the following simple example. Suppose that we have two operators A, B and a state vector ψ such as

$$A = \begin{pmatrix} a_{11} & a_{12} & 0 & 0 \\ a_{21} & a_{22} & 0 & 0 \\ 0 & 0 & a_{33} & a_{34} \\ 0 & 0 & a_{43} & a_{44} \end{pmatrix}, \quad B = \begin{pmatrix} b_{11} & b_{12} & 0 & 0 \\ b_{21} & b_{22} & 0 & 0 \\ 0 & 0 & b_{33} & b_{34} \\ 0 & 0 & b_{43} & b_{44} \end{pmatrix}, \quad \psi = \begin{pmatrix} c_1 \\ c_2 \\ 0 \\ 0 \end{pmatrix} \tag{3.19}$$

with nonzero complex numbers c_1, c_2. The GNS procedure associated to the state ψ yields

$$\pi_\omega(A) = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}, \quad \pi_\omega(B) = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}. \tag{3.20}$$

So, it can happen that $A \equiv_\psi B$ even when $A \neq B$. This result is interpreted as follows. The probability of emergence of eigenstates associated to the right-lower blocks of the matrices A and B is zero in the state ψ, and hence these right-lower blocks exhibit no measurable effects. Thus, when we are concerned with the measured values in the state ψ, it is justified to discard these irrelevant parts and to leave only the relevant parts as (3.20).

Using the Ozawa equality we can characterize the equality between the quantity to be measured indirectly and the quantity to be read out directly. The initial state vectors of the object and the apparatus are denoted as ψ and ξ, respectively. A precise measurement of the object quantity A by the meter quantity M via the measurement process α from the initial state $\nu = \psi \otimes \xi$ of the composite system is characterized by the Ozawa equality

$$A \equiv_\nu \alpha(M). \tag{3.21}$$

Then the observed values of A and $\alpha(M)$ always coincide.

3.2 Main theorem

We have introduced the three notions: isolated conservation law, covariant indicator, and Ozawa equality. Combining them we deduce the general superselection rule.

Theorem: If a measurement process α admits the isolated conservation law associated to the group action of G, and if an observable A of the object system is precisely measurable by a covariant indicator M in the sense of the Ozawa equality in an arbitrary object state ψ, then the quantity A is G-invariant. Namely, the measureable quantity must satisfy

$$\sigma_g A = A \tag{3.22}$$

for arbitrary $g \in G$. Let us rephrase the above statement; a quantity A that is measurable by a G-covariant indicator via a measurement process preserving the G-symmetry of the object system must be a G-invariant quantity. If the group action is generated by J as $\sigma_s(A) = e^{iJs/\hbar} A\, e^{-iJs/\hbar}$, then the isolated conservation law $[J, H] = 0$ implies the superselection rule $[J, A] = 0$.

Proof: Under the assumption of the theorem, we have a measurement scheme (α, A, M, ν) and a group action (G, σ, τ) that satisfy the Ozawa equality $\sigma_g A \equiv_\nu \alpha(\tau_g M)$, the covariance of the indicator $\alpha(\tau_g M) = \sigma_g(\alpha M)$, and the isolated conservation law $\sigma_g \circ \alpha = \alpha \circ \sigma_g$. Then we have

$$\sigma_g A \equiv_\nu \alpha(\tau_g M) \equiv_\nu \sigma_g(\alpha M) \equiv_\nu \alpha(\sigma_g M). \tag{3.23}$$

Note that the Ozawa equality satisfies transitivity. Since $M = \mathbf{1} \otimes M$ and $\sigma_g \mathbf{1} = \mathbf{1}$ for the identity element $\mathbf{1}$ of the object algebra,

$$\alpha(\sigma_g M) = \alpha(\sigma_g(\mathbf{1} \otimes M)) = \alpha(\mathbf{1} \otimes M). \tag{3.24}$$

Therefore, the equalities appearing in (3.23) do not depend on $g \in G$, and hence

$$\sigma_g A \equiv_\nu A \tag{3.25}$$

holds for the state vector $\nu = \psi \otimes \xi$. The operator A is defined in the Hilbert space $\mathscr{H}$ of the object system. We have assumed that the state vector $\psi \in \mathscr{H}$ can be chosen arbitrarily. (This requirement is not an excessive demand. In usual experiments, the initial state ξ of the apparatus is fixed but various initial states ψ of the object are put in.) Therefore, the equality (3.25) must hold for an arbitrary vector $\psi \in \mathscr{H}$. Thus we reach the conclusion that the operator A itself must be G-invariant:

$$\sigma_g A = A. \tag{3.26}$$

End of proof.

Here we put a comment. We proved the theorem under the assumption that A and $\alpha(M)$ are covariant and precisely equal in the sense of the Ozawa equality. We can relax this

assumption and replace it by the requirement of covariance and perfect correlation between the probability operator-valued measures (POVMs) associated to the object observable and the meter observable. Under that assumption on POVMs, we will reach an almost same conclusion as (3.26).

4 Implications

In the following we will discuss physical implications of the superselection rule from the viewpoint of measurement theory.

4.1 Wigner-Araki-Yanase-Ozawa theorem

The object system has a quantity J generating the symmetry group of the system and also has a quantity A obeying a nontrivial transformation rule under the group action. Our theorem tells that we cannot measure the quantity A via a measurement process that conserves J. By contraposition, disturbance of J is inevitable in any measurement of A. This interpretation of the theorem reminds us the uncertainty relation. Let us examine this point.

The limitation on accuracy of measurements under conservation laws is known as the Wigner-Araki-Yanase (WAY) theorem [24, 25], which states that a precise measurement of a quantity that does not commute with an additive conserved quantity is impossible. Ozawa [26] reformulated the WAY theorem and proved the quantitative relation

$$\varepsilon(A)^2 \geq \frac{|\langle [A, J_1] \rangle|^2}{4\{\sigma(J_1)^2 + \sigma(J_2)^2\}}, \tag{4.1}$$

where $\varepsilon(A) := \sqrt{\langle (\alpha(M) - A)^2 \rangle}$ is the error of measurement of A and $\sigma(J) := \sqrt{\langle (J - \langle J \rangle)^2 \rangle}$ is the standard deviation of J. The quantity J_1 belongs to the object system while the quantity J_2 belongs to the apparatus. It is assumed that their sum $J = J_1 + J_2$ is conserved during the measurement process. It is also assumed that the meter observable M commutes with J_2.

In our setting, it is assumed that the quantity J_1 alone is a conserved quantity. The quantity J_2 can be defined to be identically zero and hence we have $\sigma(J_2) = 0$. An initial state such that $\sigma(J_1) = 0$ can be prepared. If $\langle [A, J_1] \rangle \neq 0$, the error $\varepsilon(A)$ diverges and the measurement fails to make sense. This consequence resembles the superselection rule. Thus, the superselection rule can be regarded as the strongest version of the Wigner-Araki-Yanase-Ozawa theorem.

However, our derivation of the superselection rule elucidates the role of covariance and clarifies the meaning of measurability. Suppose that the transformation group of A is generated by J_1. For accomplishing a relevant measurement, it is desirable that the value of the meter $\alpha(M)$ varies when the value of the object quantity A varies. This is the requirement of covariance. But the covariant correlation cannot be made via a measurement

process that conserves J_1. In this sense, A is non-measurable. In our view, the quantity A is non-measurable not because the measurement error is infinite but because the value of the meter observable $\alpha(M)$ cannot follow the value of the object quantity A. The conservation law of J_1 obstructs making the correlation between A and $\alpha(M)$.

4.2 Charge superselection rule

A typical example of superselection rule is the charge superselection rule, which follows the $U(1)$ global symmetry. For bosonic or fermionic creation and annihilation operators $A_j^\dagger$ and A_j, the number operator

$$N := \sum_{j=1}^{n} A_j^\dagger A_j, \tag{4.2}$$

which is called the charge, generates the unitary operator

$$U_\theta := e^{iN\theta} \tag{4.3}$$

for $\theta \in \mathbb{R}$ and implements the gauge transformation

$$\sigma_\theta(B) := U_\theta B U_\theta^\dagger, \qquad \sigma_\theta(A_j) = e^{-i\theta} A_j, \qquad \sigma_\theta(A_j^\dagger) = e^{i\theta} A_j^\dagger. \tag{4.4}$$

Assume that the number operator N is an isolated conserved quantity. Namely, assume that any physically realizable measurement process α preserves $\alpha(N) = N$. Then the superselection rule tells that both the self-adjoint component $\frac{1}{2}(A_j + A_j^\dagger)$ nor the anti-self-adjoint component $\frac{1}{2i}(A_j - A_j^\dagger)$ of the annihilation operator are non-measurable. However, since the product $A_j^\dagger A_k$ is gauge invariant,

$$\frac{1}{2}(A_j^\dagger A_k + A_k^\dagger A_j), \qquad \frac{1}{2i}(A_j^\dagger A_k - A_k^\dagger A_j) \tag{4.5}$$

are measurable quantities. For superconductivity A_j represents the Cooper condensate while for superfluidity A_j represents the Bose-Einstein condensate. Although A_j itself is not measurable, a contact of two superconductors defines a measurable product $A_j^\dagger A_k$. For example, the Josephson current is a function of $A_j^\dagger A_k$, which can be interpreted as a function of the phase difference of complex Cooper condensates.

4.3 Color superselection rule

Let us discuss an implication of the superselection rule for the non-abelian gauge theory, for example, QCD. We do not attempt to provide a fully developed analysis of the color confinement problem here. We would like to ask the reader to permit the presentation of our immature idea.

The color charges are generators of the $SU(3)$ symmetry, which is the unbroken rigorous symmetry of the microscopic world. Thus, the color charges are subject to an isolated conservation law. Hence, the superselection rule tells that any measurable quantity must commute with the color charges. In other words, a measurable quantity must be colorless.

Since $SU(3)$ is a non-abelian group, the color charges themselves do not commute with each other. Therefore, the color charges are non-measurable. By the same reason, quark and gluon fields are un-observable. This is a possible explanation of the color confinement.

The color superselection rule can be compared with the charge superselection rule. The $U(1)$ symmetry of QED is also unbroken rigorous symmetry. Thus, the electric charge is an isolated conserved quantity of the microscopic world. The superselection rule tells that charged complex fields are un-observable. However, since $U(1)$ is an abelian group, the electric charge commutes with itself. Therefore, the electric charge is measurable.

As well known, quantization of fields subject to local gauge symmetry is highly nontrivial. The quantum theory of the non-abelian gauge field involves various subtle ingredients like ghost fields, auxiliary fields, an indefinite-metric space, the BRS condition, and so on. Strocchi [27] showed that every observable is a color singlet as a consequence of locality. Ojima [28] also investigated the observability condition of physical quantities using the BRS symmetry. However, we do not yet have a decisive solution of the confinement problem. Although our consideration in the present form seems not immediately applicable to the quantum theory of gauge fields, we provided a standpoint, at least, from which the confinement problem is viewed as a subject of measurement theory.

On the other hand, by the color confinement physicists usually mean not only that colored quantities are non-detectable but also that quarks and gluons are confined in hadrons. Namely, the confinement problem includes also the problem of bound states of strongly interacting particles. This aspect is a matter of genuine dynamics and is out of the scope of measurement theory.

4.4 Angular momentum

The color superselection rule can be compared with the angular momentum conservation law. Angular momenta are generators of the rotation group $SO(3)$, which is a non-abelian group. However, we can measure angular momenta of various microscopic systems; experimentalists measure angular momenta of atoms or nuclei by using the Zeeman effect or the Stern-Gerlach experiment setting. They measure also spin angular momenta of photons by using polarization filters or birefringent media. These facts give a rise of a question; why does not the superselection rule associated to the rotational symmetry prohibit measurement of angular momenta?

We can measure angular momenta because the rotational symmetry is spontaneously broken in the macroscopic world. For example, a molecule of water has a non-spherical shape. Shapes of carbohydrate molecules and protein molecules are not rotationally invariant, either. As another example, magnetization of bulk of iron breaks the rotational symmetry. In the macroscopic world, there are a lot of objects that have rotationally asymmetric shapes. Rotationally asymmetric objects can carry nonzero angular momenta. Therefore, the conservation of angular momenta is not closed in the microscopic world. An interaction can transfer angular momenta between a microscopic system and a macroscopic

system. Actually, all of the experimental settings for measuring angular momenta of microscopic systems break the rotational symmetry by applying asymmetric external fields on microscopic systems. The spontaneous breaking of the rotational symmetry allows the existence of rotationally asymmetric macroscopic objects and enables the measurements of angular momenta by macroscopic apparatus.

On the other hand, the color $SU(3)$ is not spontaneously broken. Therefore, the color superselection rule continues to hide color charges from measurements. This argument for justifying the color confinement may seem a tautology; the color is invisible from the macroscopic world because the color symmetry has not been broken spontaneously and there are no macroscopic objects that carry color charges. But the non-existence of colored objects in the macroscopic world sounds just a rephrasing of the color confinement.

However, this argument is not a tautology. Our derivation of superselection rules tells a mechanism of the superselection rules; the symmetry of measurement process prevents the indicator from moving sensitively to a change of the quantity that obeys a nontrivial transformation law under the action of the symmetry group. This also tells an approach for measuring the quantity that obeys the nontrivial transformation law; we can measure the quantity by breaking the symmetry explicitly. For example, in the Stern-Gerlach experiment setting, the spin of an atom is measured by applying an inhomogeneous magnetic field, which breaks the rotational symmetry explicitly. In the case of superconductivity, the phase of a Cooper condensate can be measured by bringing another Cooper condensate and by making a Josephson junction between them. The second condensate exchanges Copper pairs with the first one and breaks the isolated conservation law of the number of Cooper pairs of the first system. Then the relative phase of the two condensates can be measured by Josephson current. Similarly, contact of color superconducting objects will enable measurement of their relative color. In this manner, by understanding superselection rules from the viewpoint of measurement theory, we can find a method to extend the class of measurable quantities.

5 Conclusion

In this paper, we reviewed the formulation of superselection rule, which restricts the class of measurable physical quantities by requiring them to commute with the superselection charge. We examined two examples, in which the momentum conservation law and the position measurement are incompatible. In other words, the translational symmetry prevents the meter from moving covariantly to the position of the object. The analysis of these examples told a lesson that symmetry of measurement process restricts the class of feasible measurements. For accomplishing a meaningful measurement it is necessary to make a covariant correlation between the quantity of an object and the indicator of an apparatus. We introduced the three basic notions, isolated conservation law, covariant indicator, and Ozawa equality, to prove the theorem; if a measurement process preserves the symmetry

of the object system, a quantity measurable by a covariant indicator should be invariant under the symmetry group action. This theorem justified superselection rules from the viewpoint of measurement theory. The implication of the charge superselection rule was discussed. It was also argued that invisibility of colored quantities can be understood as a consequence of the non-abelian color symmetry. It was noted that angular momenta are also conserved quantities associated to the non-abelian $SO(3)$ symmetry but they are measurable in experiments. Spontaneous breaking of the rotational symmetry allows existence of rotationally asymmetric macroscopic objects that can exchange angular momenta with microscopic objects. Thus, conservation of angular momenta is not closed in the microscopic world. Hence the superselection rule is not applicable to the rotational symmetry and we can measure microscopic rotational variables, in particular, a spin component of a particle. This consideration suggests a method for measuring a quantity that obeys a nontrivial transformation law under symmetry operations; the indicator can move covariantly to the object quantity if we use a measurement process which breaks the symmetry, for example, by applying external field or by bringing another subsystem and allowing an interaction that breaks isolation of the object. These considerations are not just rephrasing of superselection rules; they tell a mechanical foundation of superselection rules and also a method for overcoming superselection rules.

We showed that the isolated conservation law defines a superselection rule and hence defines the class of microscopic quantities that are measurable by outside observers. It may be possible to say that the isolated conservation law defines a border between the microscopic object world and the macroscopic observer world. The degree of isolation of the object system is variable. Thus the class of measurable quantities can vary depending on available measurement interactions. These considerations tell that the superselection rules are not absolute rules.

Finally, we would like to mention a view of the physical world brought by the study of superselection rules. Nature has a hierarchical structure like quarks, hadrons, atoms, molecules, polymers, condensed matters, cells, life, and so on. It can be said that the hierarchical structure is based on nesting of isolations. The conservation laws of colors, quark flavors, lepton flavors, chiral symmetry, isospins, electric charges, angular momenta, and linear momenta define various levels of isolations. For example, the colors are isolated and conserved in hadrons, the isospins are isolated in nuclei, and so on. Some of them are rigorous unbroken symmetries, some are broken, and the others are approximate symmetries. The scales of symmetry breakings also spread over from the electroweak scale to the molecular scale. Each level of isolated symmetry or broken symmetry corresponds each hierarchy of nature. In this view, it is recognized that the division between the micro and the macro is not fixed but there are various micro-macro strata which are marked by the superselection rules. This view seems in harmony with Anderson's view that symmetry breakings generate each hierarchy of nature [29] and Ojima's concept of the Micro-Macro duality on the bi-directional functions of the micro and the macro physics [5].

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